Complex Number System

There is no real number x that satisfies the Polynomial equation $x^2+1=0$ To permit solutions of this and similar equations, the set of complex number is introduced.

A complex number as having the form $a+bi$ where a and b are real numbers and i, which is called the imaginary unit, has the property that $i^2=-1$
If $z=a+ib$ then a is called the real part of z and b is called the imaginary part of z and are denoted by $Re(z)$ and $Im(z)$ respectively. The symbol z which can stand for any C.N. is called complex variable.

FUNDAMENTAL OPERATIONS WITH COMPLEX NUMBERS

I Addition

$$(a+bi)+(c+di)$$

$$\Rightarrow (a+c)+i(b+d)$$

2. SUBTRACTION – $(a+bi)-(c+di)$

$$=(a-c)+(b-d)i$$

3. MULTIPLICATION

$$(a+bi)(c+di)$$

$$= ac+adi+bci+bdi^2$$

$$=(ac-bd)+(ad+bc)i$$

4.) Division If $c \neq 0$ and $d \neq 0$

then $\dfrac{a+bi}{c+di} = \dfrac{a+bi}{c+di} \cdot \dfrac{c-di}{c-di}$

$$= \frac{ac-adi+bci-bdi^2}{c^2-d^2i^2}$$

$$= \frac{ac+bd+(bc-ad)i}{c^2+d^2}$$

$$= \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2} \cdot i$$

ABSOLUTE VALUE

The absolute value or modulus of a complex number $a+bi$ is defined as $|a+bi| = \sqrt{a^2+b^2}$

Example 1.1 $|-4+2i| = \sqrt{(-4)^2+2^2} = \sqrt{20}$
$= 2\sqrt{5}$

If $z_1, z_2 \ldots\ldots z_m$ are complex numbers, the following properties hold

1. $|z_1 z_2| = |z_1||z_2|$
   or $|z_1 z_2 \ldots\ldots z_m| = |z_1||z_2| \ldots |z_m|$

2. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ if $z_2 \neq 0$

3. $|z_1 \pm z_2| \leq |z_1| + |z_2|$

4. $|z_1 \pm z_2| \geq |z_1| - |z_2|$